Differential Structure on κ-Minkowski Spacetime Realized as Module of Twisted Weyl Algebra

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The differential structure on the κ-Minkowski spacetime from Jordanian twist of Weyl algebra is constructed, and it is shown to be closed in 4-dimensions in contrast to the conventional formulation. Based on this differential structure, we have formulated a scalar field theory in this κ-Minkowski spacetime.

I. INTRODUCTION

κ-Minkowski spacetime consists of the noncommutative coordinates satisfying

$$[x^0 \overset{*}{,} x^i] \equiv x^0 * x^i - x^i * x^0 = \frac{i}{\kappa} x^i. \tag{1}$$

It was first introduced as translations in κ-Poincaré group, dual to the κ-Poincaré algebra [1, 2]. It has attracted much interests as a realization of the so-called doubly special relativity (also known as deformed special relativity) [3]. The differential structure [4] and scalar field theory [5, 6, 7, 8, 9] have been formulated in this κ-Minkowski spacetime. Various physical implications have been investigated [10, 11]. It is, however, known that the differential structure for this κ-Minkowski spacetime leads to the momentum space corresponding to a de Sitter section in five dimensional flat space, and that there exist complex poles in the free scalar field propagator, which implies the existence of unphysical ghost modes in this formulation [5].

To circumvent the difficulties in understanding the physical implications of the five dimensional differential structure and the existence of complex modes in the free theories, the κ-Minkowski spacetime from twist deformation of underlying symmetry of the spacetime were formulated [12]. The virtue of the twist formulation is that the deformed symmetry algebra is the same as the original undeformed one and only the coproduct structure changes, leading to the same free field structure as the corresponding commutative field theory. Such twist formulation of noncommutative field theories was successfully applied to the case of the canonical noncommutative spacetime [13].

The attempt to realize the κ-Minkowski spacetime by twisting Ponicaré algebra is successful only for the light-cone κ-Minkowski spacetime [14]. The realization of the time-like κ-Minkowski spacetime by twist was succeeded recently by enlarging the symmetry algebra of the spacetime to $igl(4, R)$ by the authors of [15] and [16]. The corresponding differential structure was constructed in [17], which was shown to be closed in 4-dimensions contrary to the case of the conventional formulation which needs an extra fifth dimension. Based on this differential structure, scalar field theory is formulated in [18]. Some physical properties of this twist realization was also discussed in [19].

The twist realization of the κ-Minkowski spacetime is also possible by twisting the Weyl and conformal algebras, which are smaller than $igl(4, R)$ and physically relevant, by using the Jordanian twist [20, 21, 22, 23]. One may also consider the chains of twists for classical Lie algebras [24]. It is the purpose of this paper to construct the differential structure of κ-Minkowski spacetime obtained by the Jordanian twist of the Weyl algebra, and to construct a free scalar field theory by using this twist approach.

In sec. II, we give a brief review of the Jordanian twist deformation of Weyl algebra, and find the corresponing $*$-product from the twist. We construct the differential structure of the κ-Minkowski spacetime, and find that it is closed in 4-dimensions in sec. III. In sec. IV, we construct a scalar field theory action, and discuss the physical implications of the theory in the last section.

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II.   TWISTED WEYL (HOPF) ALGEBRA

In this section, we review the twist deformation of Weyl algebra by using the Jordanian twist [12, 20, 21]. Given a Hopf algebra, one can define another (deformed) Hopf algebra using twist element $\mathcal{F}$ which obeys

$$(\mathcal{F} \otimes 1)(\triangle \otimes id)\mathcal{F} \;=\; (1 \otimes \mathcal{F})(id \otimes \triangle)\mathcal{F}, \tag{2}$$

$$(\epsilon \otimes 1)\mathcal{F} \;=\; (1 \otimes \epsilon)\mathcal{F}. \tag{3}$$

The product and the counit of the deformed Hopf algebra do not change, but the coproduct is deformed according to

$$\triangle_t Y = \mathcal{F} \triangle_0 Y \mathcal{F}^{-1}, \tag{4}$$

where Y represents generators of the original algebra and $\triangle_0 Y = Y \otimes 1 + 1 \otimes Y$. Along with this deformation, module algebra must also be deformed for the algebra to act covariantly, that is, the action of the generators of twist deformed Hopf algebra must be of the same form as that of the original algebra

$$Y \rhd m(f \otimes g) = m(\triangle Y \rhd f \otimes g) = m(Y^{(1)} \rhd f \otimes Y^{(2)} \rhd g), \tag{5}$$

where m denotes the product of the module algebra and we use the Swedler notation $\triangle Y = Y^{(1)} \otimes Y^{(2)}$. The deformed module algebra is defined by replacing the product m by the new $*$-product, m_*,

$$f * g = m_*(f \otimes g) = \mu(\mathcal{F}^{-1} \rhd f \otimes g). \tag{6}$$

We can apply this deformation to Weyl (Hopf) algebra, which is generated by 11 generators, P_0, P_i, M_i, N_i, and D, each correspoding to those of time translation, space translation, rotation, boost and dilatation, respectively. These generators satisfy [25]

$$\begin{array}{ll}
[D, P_\mu] = iP_\mu, & [D, M_i] = 0 = [D, N_i], \\
[P_0, M_i] = 0, & [P_0, N_i] = iP_i, \\
[P_i, M_j] = i\epsilon_{ijk}P_k, & [P_i, N_j] = -i\eta_{ij}P_0, \\
[M_i, M_j] = i\epsilon_{ijk}M_k, & [M_i, N_j] = i\epsilon_{ijk}N_k, \\
[N_i, N_j] = -i\epsilon_{ijk}M_k. &
\end{array} \tag{7}$$

The greek indices μ and ν run over 0, 1, 2, 3, Latin indices i, j and k run over 1, 2, 3, and $\eta_{\mu\nu} = \mathrm{diag}(+1, -1, -1, -1)$. The repeated index implies the summation over that index.

Weyl algebra has a representation on the space of coordinates x^μ:

$$\begin{array}{ll}
D \rhd x^\rho = -ix^\rho, & P_\mu \rhd x^\rho = -i\eta_\mu^\rho, \\
M_i \rhd x^0 = 0, & M_i \rhd x^l = -i\epsilon^l_{im}x^m, \\
N_i \rhd x^0 = -i\eta_{i\rho}x^\rho, & N_i \rhd x^l = i\eta^l_i x^0.
\end{array} \tag{8}$$

As a twist element, we choose

$$\mathcal{F} = \exp\left(-iD \otimes \ln\left(1 + \frac{P_0}{\kappa}\right)\right), \tag{9}$$

which satisfies Eq. (2) and known as a Jordanian twist [20, 21, 22]. Coproduct of the twisted Weyl algebra is obtained from Eq. (4):

$$\triangle_t D \;=\; 1 \otimes D + D \otimes \frac{1}{1 + \frac{P_0}{\kappa}}, \tag{10}$$

$$\triangle_t P_\mu \;=\; P_\mu \otimes \left(1 + \frac{P_0}{\kappa}\right) + 1 \otimes P_\mu, \tag{11}$$

$$\triangle_t M_i \;=\; M_i \otimes 1 + 1 \otimes M_i, \tag{12}$$

$$\triangle_t N_i \;=\; N_i \otimes 1 + 1 \otimes N_i + D \otimes \frac{\frac{P_i}{\kappa}}{1 + \frac{P_0}{\kappa}}. \tag{13}$$

In twisted Weyl algebra, the spatial rotations are undeformed, thereby retaining the rotational symmetry of the 3-dimensional space.

As a module algebra, the space of the functions also has to be deformed. The product of this algebra is replaced by the $*$-product defined by Eq. (6). Using this $*$-product, the product of two coordinates becomes

$$x^\mu * x^\nu = x^\mu x^\nu - \frac{i}{\kappa}\eta^{0\nu}x^\mu, \tag{14}$$

and the commutation relations between coordinates become

$$[x^0 \overset{*}{,} x^i] \equiv x^0 * x^i - x^i * x^0 = \frac{i}{\kappa}x^i, \quad [x^i \overset{*}{,} x^j] = 0, \tag{15}$$

which is the commutation relation for κ-Minkowski spacetime, Eq. (1).

For later convenience, we calculate the $*$-product of two exponential functions,

$$e^{ip\cdot x} * e^{iq\cdot x} = e^{i(p+q+\frac{q_0}{\kappa}p)\cdot x} \tag{16}$$

where $p \cdot x = p_\mu x^\mu$. As expected, this implies the addition of momenta described by the coproduct of P_μ, Eq. (11). This $*$-product corresponds to the time-left ordering,

$$: e^{ip\cdot x} := e^{ip_0 x^0} * e^{ip_i x^i}, \tag{17}$$

of the exponential kernel function in the conventional approach of the κ-Minkowski spacetime.

In the case of twist deformation of $igl(4, R)$ [15, 16], there exists a free parameter in the twist element. Depending on the value of the parameter, the resultant $*$-product corresponds to a specific ordering prescription of the exponential function in the conventional formulation. There also exists such a freedom in the case of the Jordanian twist. If one chooses the twist element,

$$\mathcal{F}_2 = \exp\left(-i\ln\left(1 - \frac{P_0}{\kappa}\right) \otimes D\right), \tag{18}$$

instead of (9), the resultant $\star$-product becomes

$$e^{ip\cdot x} \star e^{iq\cdot x} = e^{i(p+q-\frac{p_0}{\kappa}q)\cdot x}, \tag{19}$$

which corresponds to the time-right ordering,

$$: e^{ip\cdot x} := e^{ip_i x^i} \star e^{ip_0 x^0}, \tag{20}$$

in the conventional formulation.

It is noted that with the twist (9) we have

$$x^0 * x^0 = (x^0)^2 - \frac{i}{\kappa}x^0 \neq (x^0)^2,$$

and

$$e^{ip_0 x^0} * e^{iq_0 x^0} = e^{i(p_0+q_0+\frac{p_0 q_0}{\kappa})x^0} \neq e^{i(p_0+q_0)x^0},$$

in contrast to the conventional formulation and the $igl(4, R)$ twist formulation.

III. DIFFERENTIAL STRUCTURE

In Ref. [4], a bicovariant differential calculus [26] was constructed on the κ-Minkowski spacetime based on the κ-Poincaré algebra. In the case of this conventional κ-Minkowski spacetime based on the κ-Poincaré algebra, it turns out that there does not exist 4-D differential calculus which is Lorentz covariant, but one needs to introduce an extra fifth dimension.

In a similar way, we obtain the differential calculus on κ-Minkowski spacetime of the last section. The external derivative d is demanded to satisfy the Leibniz rule:

$$d(f * g) = df * g + f * dg. \tag{21}$$

For the generators Y of the Weyl algebra, the action on the differential algebra may be postulated as

$$
\begin{aligned}
Y \triangleright dx^\mu &= d(Y \triangleright x^\mu), \\
Y \triangleright x^\mu * dx^\nu &= (Y^{(1)} \triangleright x^\mu) * (Y^{(2)} \triangleright dx^\nu).
\end{aligned}
$$

For the twisted Weyl algebra, we obtain the following identities from the representation (8):

$$
\begin{aligned}
D \triangleright [x^0 \overset{*}{,} dx^\mu] &= -2i[x^0 \overset{*}{,} dx^\mu] - \frac{1}{\kappa} dx^\mu, \\
D \triangleright [x^l \overset{*}{,} dx^\mu] &= -2i[x^l \overset{*}{,} dx^\mu], \\
M_i \triangleright [x^0 \overset{*}{,} dx^0] &= 0, \\
M_i \triangleright [x^0 \overset{*}{,} dx^l] &= i\epsilon^l{}_{im}[x^0 \overset{*}{,} dx^m], \\
M_i \triangleright [x^l \overset{*}{,} dx^0] &= i\epsilon^l{}_{im}[x^l \overset{*}{,} dx^m], \\
M_i \triangleright [x^l \overset{*}{,} dx^m] &= i\epsilon^m{}_{in}[x^l \overset{*}{,} dx^n] + i\epsilon^l{}_{in}[x^n \overset{*}{,} dx^m], \\
N_i \triangleright [x^0 \overset{*}{,} dx^0] &= -i\eta_{il}[x^l \overset{*}{,} dx^0] - i\eta_{il}[x^0 \overset{*}{,} dx^l], \\
N_i \triangleright [x^0 \overset{*}{,} dx^l] &= -i\eta_{im}[x^m \overset{*}{,} dx^l] + i\eta^l{}_i[x^0 \overset{*}{,} dx^0], \\
N_i \triangleright [x^l \overset{*}{,} dx^0] &= i\eta^l{}_i[x^0 \overset{*}{,} dx^0] - i\eta_{im}[x^l \overset{*}{,} dx^m] \\
&\quad + \frac{1}{\kappa}\eta^l{}_i dx^0, \\
N_i \triangleright [x^l \overset{*}{,} dx^m] &= i\eta^l{}_i[x^0 \overset{*}{,} dx^m] + i\eta^m{}_i[x^l \overset{*}{,} dx^0] \\
&\quad + \frac{1}{\kappa}\eta^l{}_i dx^m.
\end{aligned}
$$

We find that these identities are satisfied if we demand

$$
\begin{aligned}
\left[x^0 \overset{*}{,} dx^\mu\right] &= \frac{i}{\kappa} dx^\mu, \\
\left[x^i \overset{*}{,} dx^\mu\right] &= 0,
\end{aligned}
\tag{22}
$$

which is similar to the differential structure of $igl(4, R)$ twist formulation, except that $[x^0 \overset{*}{,} dx^0]$ does not vanish. If one tries similar differential structure in the κ-Minkowski spacetime based on the κ-Poincaré algebra as (22), they do not satisfy the mixed Jacobi identity [4]. However, it is easy to show that the commutation relations (22) is consistent with the mixed Jacobi identity:

$$
[x^\mu \overset{*}{,} [x^\nu \overset{*}{,} dx^\rho]] + [x^\nu \overset{*}{,} [dx^\rho \overset{*}{,} x^\mu]] + [dx^\rho \overset{*}{,} [x^\mu \overset{*}{,} x^\nu]] = 0,
\tag{23}
$$

and the commutation relations for the κ-Minkowski spacetime, Eq. (15).

Eq. (22) defines the differential structure of the κ-Minkowski spacetime from the Jordanian twist of Weyl algebra. Contrary to the conventional κ-Minkowski spacetime in which only covariant 5-D differential calculus exists, the differential calculus is closed in 4-dimensions and is covariant under the twisted Weyl algebra.

We may define partial derivative ∂_μ as

$$
df = \partial_\mu f * dx^\mu.
\tag{24}
$$

To find the properties of the partial derivative, it is sufficient to examine the derivatives of the exponential function. From Eq. (21) and Eq. (22) we find

$$
dx^\mu * e^{iq \cdot x} = \left(1 + \frac{q_0}{\kappa}\right) e^{iq \cdot x} * dx^\mu,
\tag{25}
$$

and

$$
de^{ip \cdot x} = ip_\mu e^{ip \cdot x} * dx^\mu.
\tag{26}
$$

Comparing Eq. (24) and Eq. (26), we find that ∂_μ acts like an ordinary partial derivative,

$$
\partial_\mu e^{ip \cdot x} = ip_\mu e^{ip \cdot x}.
\tag{27}
$$

However, this partial derivative ∂_μ does not obey the ordinary Leibniz rule, but satisfies

$$
\partial_\mu(f * g) = \partial_\mu f * g + f * \partial_\mu g - \frac{i}{\kappa}\partial_\mu f * \partial_0 g.
\tag{28}
$$

IV. ACTION FOR SCALAR FIELD THEORY

In this section we first consider the Fourier transfomation of a scalar field, define an adjoint derivative, and then write down the action for the massless scalar field theory invariant under the twisted Weyl algebra.

We define the Fourier transformation of a scalar field as

$$\phi(x) \;=\; \int d\mu(p) e^{ip\cdot x}\tilde{\phi}(p), \tag{29}$$

where $d\mu(p)$ is the integration measure to be determined. The inverse Fourier transformation has to be defined using $*$-product and the conjugate of exponential function. We demand that the conjugate of the exponential function obeys the relations,

$$(e^{ip\cdot x} * e^{iq\cdot x})^{\dagger} \;=\; (e^{iq\cdot x})^{\dagger} * (e^{ip\cdot x})^{\dagger},$$
$$e^{ip\cdot x} * (e^{ip\cdot x})^{\dagger} \;=\; 1 = (e^{ip\cdot x})^{\dagger} * e^{ip\cdot x},$$
$$((e^{ip\cdot x})^{\dagger})^{\dagger} \;=\; e^{ip\cdot x},$$

and find that these relations are satisfied if we define the conjuagation of exponetial function as

$$(e^{ip\cdot x})^{\dagger} = \exp\left(i\left(-\frac{p_\mu}{1 + \frac{p_0}{\kappa}} \right) x^\mu \right). \tag{30}$$

From this, we define the deformed antipode S_t of P_μ as in Ref. [8]:

$$S_t(P_\mu) = \frac{-P_\mu}{1 + \frac{P_0}{\kappa}}. \tag{31}$$

We find that this definition obeys the relation,

$$\cdot (S \otimes id)\triangle = \cdot (id \otimes S)\triangle = \eta\epsilon, \tag{32}$$

for the antipode [12] and is the same as that in [21].

Using this conjugate of exponential function, we define the delta function:

$$\int_x (e^{ip\cdot x})^{\dagger} * e^{iq\cdot x} = (2\pi)^4 \left(1 + \frac{p_0}{\kappa} \right) \delta^{(4)}(q - p), \tag{33}$$

where $\int_x \equiv \int d^4x$. We now can define the inverse Fourier transformation as

$$\tilde{\phi}(p) \;=\; \int_x (e^{ip\cdot x})^{\dagger} * \phi(x). \tag{34}$$

From the definition of Fourier transformation and the delta function, we find the invariant measure

$$d\mu(p) = \frac{d^4p}{(2\pi)^4 \left(1 + \frac{p_0}{\kappa} \right)}. \tag{35}$$

Adjoint derivative $\partial^{\dagger}$ is defined as

$$\int_x \phi(x) * \partial_\mu \phi(x) = \int \partial_\mu^{\dagger} \phi(x) * \phi(x). \tag{36}$$

It can be shown that

$$\partial_\mu^{\dagger} e^{ip\cdot x} = i\left(-\frac{p^\mu}{1 + \frac{p_0}{\kappa}} \right) e^{ip\cdot x}, \quad (\partial_\mu e^{ip\cdot x})^{\dagger} = -\partial_\mu^{\dagger}(e^{ip\cdot x})^{\dagger}. \tag{37}$$

The action for massless scalar field can be written in analogy with that of the commutative spacetime as

$$S = \int_x (\partial_\mu \phi(x))^{\dagger} * \partial^\mu \phi(x). \tag{38}$$

Using the Fourier transformation defined above, we write the action in momentum space

$$S = \int_p \tilde{\phi}^{\dagger}(p)\, p_\mu p^\mu\, \tilde{\phi}(p), \tag{39}$$

which is consistent with the dispersion relation discussed in [21]. Thus it implies that in momentum space the free field structure of the κ-Minkowski spacetime is the same as that of the corresponding commutative theory except for the measure factor as expected in the twist formulation.

V. DISCUSSION

We have constructed a differential structure on κ-Minkowski spacetime from twisting the Weyl symmetry group [21], and have shown that the differential structure is closed in 4-dimensions as in the case of the twist formulation based on $IGL(4, R)$ group [17]. Based on this differential structure, we have formulated a free scalar field theory in this noncommutative spacetime. As in the case of the $IGL(4, R)$ twist, the dispersion relation is the same as the correponding commutative theory, and the theory has the same free field structure as in the commutative case. Thus one may avoid various difficulties encountered in the conventional formulation of the κ-Minkowski spacetime based on the κ-Poincaré algebra, and study the effects of the κ-deformation more easily using this twist formulation.

It is noted that, contrary to other approaches, we have $x^0 * x^0 = (x^0)^2 - \frac{i}{\kappa}x^0$ and $[x^0 \overset{*}{,} dx^0] = \frac{i}{\kappa}dx^0$. One may relate the action of translation generator P_μ on the space of functions to the external derivative d as

$$df = (iP_\mu \triangleright f) * dx^\mu. \tag{40}$$

With this relation, Eq. (28) is consistent with the action on the deformed space of functions of coordinates for translation generator P_μ of the twisted Weyl algebra, whose coproduct is given by Eq. (11),

$$\partial_\mu(f * g) = iP_\mu \triangleright m_*(f \otimes g). \tag{41}$$

It is noted that the external derivative d satisfies the Leibniz rule even though partial derivative ∂_μ does not. This comes from the commutation relation between dx^μ and the exponential function, Eq. (25):

$$
\begin{aligned}
d(f * g) &\equiv \partial_\mu(f * g) * dx^\mu \\
&= \left[\partial_\mu f * \left(g - \frac{i}{\kappa}\partial_0 g\right)\right] * dx^\mu + (f * \partial_\mu g) * dx^\mu \\
&= (\partial_\mu f * dx^\mu) * g + f * (\partial_\mu g * dx^\mu) \\
&= df * g + f * dg.
\end{aligned}
$$

As in other twist formulations, the intrinsic effects of such κ-deformation would appear as a result of interactions. As an interaction in the Weyl symmetric theories one would naturally consider the ϕ^4-interaction,

$$\frac{\lambda}{4!}\int_x \phi(x) * \phi(x) * \phi(x) * \phi(x). \tag{42}$$

This interaction term becomes, in momentum space,

$$\frac{\lambda}{4!}(2\pi)^4 \int_{klpq} \tilde{\phi}(k)\tilde{\phi}(l)\tilde{\phi}(p)\tilde{\phi}(q)\delta(k_{lpq} + l_{pq} + p_q + q), \tag{43}$$

where $k_{lpq} = k(1 + \frac{l_0}{\kappa})(1 + \frac{p_0}{\kappa})(1 + \frac{q_0}{\kappa})$, $l_{pq} = l(1 + \frac{p_0}{\kappa})(1 + \frac{q_0}{\kappa})$, $p_q = p(1 + \frac{q_0}{\kappa})$.

By change of variables, $p' = S_t(p) = -\frac{p}{1+\frac{p_0}{\kappa}}$, $q' = S_t(q) = -\frac{q}{1+\frac{q_0}{\kappa}}$, using the deformed antipode S_t of P_μ, we can rewrite Eq. (43) as

$$
\begin{aligned}
&\frac{\lambda}{4!}(2\pi)^4 \int_{klpq} \tilde{\phi}(k)\tilde{\phi}(l)\bar{\phi}(p)\bar{\phi}(q) \\
&\times \delta\left(\left(k\left(1 + \frac{l_0}{\kappa}\right) + l\right) - \left(q\left(1 + \frac{p_0}{\kappa}\right) + p\right)\right),
\end{aligned}
\tag{44}
$$

where $\bar{\phi}(q) = \frac{\tilde{\phi}(S_t(q))}{\left(1+\frac{q_0}{\kappa}\right)^{-1}}$. If we interpret this interaction term as scattering process of two incoming particle $\bar{\phi}$ into two outgoing $\tilde{\phi}$, then the delta-function in the integrand guarantees that the total momentum of in-particles is equal to that of out-particles.

It would be interesting to investigate the physical effect of this interaction and study the possibility of describing the Planck scale physics by such κ-deformation of spacetime.

For Weyl symmetry, we have to consider only the massless theory. It would be interesting to investigate the possibility of Weyl symmetry breaking through quantum corrections.

Acknowledgments

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